TEACHING DOSSIER

Cashous Bortner

Below, you will find various evaluations I have received and sample teaching materials I have used as a graduate student at North Carolina State University (NCSU), and as an undergraduate at the University of Nebraska-Lincoln (UNL).

TEACHING EVALUATIONS

North Carolina State University (Graduate Teaching Assistant).

First, pages 2-4 consist of an evaluation done by Professor Jo-Ann Cohen of NCSU as my mentor in the Preparing the Professoriate program. She observed over ten of my (virtual, synchronous) *Foundations of Advanced Mathematics* (NCSU MA 225) classes in which I was the instructor of record in the Spring of 2021, and answered the questions according to her observations. This upper-division course is the first proofs-based courses that NCSU mathematics and mathematics education students take.

In pages 5-7, NCSU Senior Lecturer Elizabeth Dempster evaluates one of my *Calculus for Life and Management Sciences A* (NCSU MA 131) classes in which I was the instructor of record in the Summer of 2019. At the time, she was my teaching mentor for this course.

Page 8 consists of the course evaluation statistics of every course that I have taught in the Mathematics Department in some capacity at NCSU. This chart shows my evaluation in **green** and the departmental averages for that term directly to the right in **light blue**. Pages 9-11 are selected comments from student course evaluations for the three of the courses in which I was an instructor of record in the Mathematics Department, followed by the courses for which I was a recitation instructor.

University of Nebraska-Lincoln (Undergraduate Teaching Assistant).

Pages 12-13 are student course evaluations for my *Calculus II* (UNL MATH107) recitation in the Fall of 2016.

Pages 14-15 are student course evaluations for my *Calculus I* (UNL MATH106) recitation in the Spring of 2016.

Pages 16-17 are student course evaluations for my *Calculus I* (UNL MATH106) recitation in the Fall of 2015.

SAMPLE TEACHING MATERIALS

Pages 18-23 consist of my syllabus for an *Foundations of Advanced Mathematics* (NCSU MA 225) course at NCSU in which I was instructor of record during the Spring of 2021.

Pages 24-26 consist of the third exam that I wrote for the *Foundations of Advanced Mathematics* (NCSU MA 225) course at NCSU in which I was instructor of record during the Spring of 2021.



Certificate of Accomplishment in Teaching Program Classroom Observation Form – Option 1

CoAT Participant's Name: Cashous (Cash) Bortner Course observed: MA 225 (Foundations of Advanced Mathematics) Date of observation: I have observed Cash more than 10 times. Below is a summary of those observations. Time of class: MWF 11:45am-12:35pm Observed by: Jo-Ann Cohen

Teaching characteristics – Comments

1. Planning and start of session

Appropriateness of aims and outcomes (where it is possible to evaluate this). Communication of these to students. Continuity with other sessions and students' prior knowledge made explicit. Coping with any unexpected occurrences, e.g. latecomers, missing equipment.

MA 225 has many learning outcomes, the most important of which are: (i) the ability to write clear, correct, and convincing proofs appropriate for the audience, (ii) the ability to analyze and critique mathematical arguments, and (iii) the ability to make conjectures and to determine the validity of those conjectures. Cash weaves 1-3 of those goals into each session of the class.

He provides the mathematical foundation necessary to understand the material in each class and links that foundation to previous sessions of MA 225 as well as to previous courses the students have taken.

2. Presentation

Structure. Relevance and organization of content. Attitude to subject matter. Clarity of presentation. Emphasis of key points. Pace of session (time management). Tone, volume, clarity of speech. Links made to other aspects of course (e.g. lectures, seminars or tutorials). Summary (end and/or interim).

Cash is well organized. He provides pdfs to the students on the course Moodle site before each class and annotates the pdfs in class using Goodnotes.

Cash explains things really well and uses the technology effectively. The pace of the course is

Form adapted with permission from: Educational Development Centre, Royal Holloway, University of London, Certificate in Academic Practice in Teaching and Learning (CAPITAL) Programme.

appropriate. He clearly displays, and conveys, his enthusiasm for mathematics in every session with the students.

3. Student participation

Question and answer technique. Exercises/activities. Class management (appropriate level of control and authority). Instructions to students. General class atmosphere. Level of participation between students (excessive? lacking?). Attention and interest. Attitude to students. Awareness of individual needs. Student-teacher rapport.

Cash engages the students through the use of breakout rooms and in-class discussions. His class is very welcoming and inclusive. I like the fact that he often tells the students how he struggled with some of the concepts. It makes him more relatable. Moreover, his delivery is appropriate for both visual and verbal learners.

I interviewed the students after one of my observations and the students were extremely positive about how Cash's style has enhanced their learning of the material.

4. Methods and approaches

Choice/variety of teaching/learning methods. Use and design of instructional materials (board, technology, handouts etc.) Use of appropriate reinforcement. Examples and analogies. References and links to research, other resources. Handling problems/disruptions.

As I said, Cash's methods are appropriate for both visual and verbal learners. He uses the Zoom technology and Moodle resources well.

One thing that I particularly like is his use of study guides. Before each test he has the students upload a study guide that they have developed and that they may use on the test. It's an assignment that I plan on using in my MA 225 class in Fall 2021.

Cash's presentations and teaching methods are characteristic of a much more experienced MA 225 instructor!

5. General

Were the aims and outcomes achieved? Appropriateness of teaching/learning methods. Was effective communication achieved? Awareness of needs of learners and differences in approach.

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Form adapted with permission from: Educational Development Centre, Royal Holloway,

We originally developed 225 to serve as a bridge between the calculus sequence and the upper level required courses for our mathematics majors. Cash does a great job of preparing students for our upper level theoretical courses.

I would like to add a few more general comments.

Cash is very aware that these are difficult times for students and I like the fact that he has made it clear to his students that he is here for them in case they want to discuss mathematical issues or other issues affecting them. I know that some students have, in fact, taken him up on his offer to be a resource for them.

For so many reasons, Cash will be a great faculty member at his next institution. We're lucky to have him here as a graduate student working with our undergraduate students at this point in his academic career!

6. Aspects to improve

Comment in terms of both teaching style and content (if possible).

My only suggestions are as follows:

- (i) I would think about pushing the students to turn on their videos in Zoom so that they are more accountable. I think that keeps them engaged in a different way, but also I know that different instructors do not feel comfortable doing that.
- (ii) Cash has indicated that his handwriting can be messy (as can mine!). So I would suggest reviewing the pdfs before posting them after class to make some of the notes more legible/less messy.

Signed by observer:

Jo-an Cohen

Date: April 16, 2021

Google Forms

Thanks for filling out <u>Teaching Assistant Evaluation Form</u> Here's what we got from you: EDIT RESPONSE

Teaching Assistant Evaluation Form

Your email address (<u>ejdempst@ncsu.edu</u>) was recorded when you submitted this form.

Name of TA

Cashous Bortner

Semester of evaluation

Summer II

Class the TA taught/helped with

Math 131

Role of TA

- Instructor
- Recitation Leader
- Lecture Assistant
- Grader

TA Instructor Evaluation Form

Date of observation

Tuesday, July 16

Approximate size of class

7

The instructor

| | Extremely well | Very well | Adequately | Inadequately | Very Poorly | Did not observe |
|--|----------------|-----------|------------|--------------|----------------|--------------------|
| Was prepared for class | • | 0 | 0 | 0 | 0 | 0 |
| Presented the material effectively | ۲ | 0 | 0 | 0 | 0 | 0 |
| Was enthusiastic about the material | • | 0 | 0 | 0 | 0 | 0 |
| Spoke clearly and audibly | ۲ | 0 | 0 | 0 | 0 | 0 |
| Made effective use of the board or other visual display | | 0 | 0 | 0 | 0 | 0 |
| Encouraged student involvement | ۲ | 0 | 0 | 0 | 0 | 0 |
| Treated students respectfully | ۲ | 0 | 0 | 0 | 0 | 0 |

Please comment on the instructors strengths or weaknesses.

Cash did an awesome job teaching the summer session class. It was a long class but he paced his lecture perfectly well, giving them time to work and think and ask questions as he presented the materials. He wrote on the board and faced the students lots, encouraging them to ask questions and be involved (which they were!) His speech was clear and at a good pace. The students seem very comfortable with him and he engaged with them in a professional, respectful way. The lecture was also very well thought out. He outlined what he was going to do and reminded the students of review materials while he worked current problems. He picked good problems in a very good way; progressively going from easiest to hardest while encouraging them to try before giving up. I liked how he encouraged them to just "doodle", think about what you know, keep writing down what you do know when he gave them the hardest application problem to try. Cash seems to be a naturally good lecturer. I think the students really liked his style of teaching and enjoyed having someone willing to break down difficult problems and help them to succeed. I think he is a superior teaching assistant for the Math department.

Would you recommend the instructor to teach this class again?

Yes!

Create your own Google Form

| | Respon | Response options: 5 = "Strongly Agree", 4 = "Agree", 3 = "Neither Agree/Disagree", 2 = "Disagree", 1 = "Strongly Disagree" | | | | | | | | |
|--|---------------|--|-----------------|---|-----------------|--|---|-------|--|--|
| [Term] [Course] ([Assignment]) | Sunne Alt sat | Stand nesonal Hatt | Sunne and state | st sidence od parts et of person of the state of the stat | Sping 2019 MANS | is and successful with the | standarden and and and and and and and and and an | erate | | |
| The instructors teaching aligned with the courses learning objectives/outcomes | 5 | 4.7 | 5 | 4.5 | 5 | 4.9 | 4.5 | | | |
| The instructor was receptive to students outside the classroom | 5 | 4.6 | 5 | 4.4 | 5 | 4.8 | 4.4 | | | |
| The instructor explained material well. | 5 | 4.4 | 4.8 | 4.2 | 4.9 | 4.9 | 4.1 | | | |
| The instructor was enthusiastic about teaching the course | 5 | 4.6 | 5 | 4.4 | 5 | 4.9 | 4.4 | | | |
| The instructor was prepared for class | 5 | 4.7 | 5 | 4.5 | 5 | 4.9 | 4.5 | | | |
| The instructor gave useful feedback | 5 | 4.4 | 5 | 4.2 | 5 | 4.9 | 4.2 | | | |
| The instructor consistently treated students with respect | 5 | 4.8 | 4.8 | 4.6 | 5 | 4.9 | 4.5 | | | |
| Overall, the instructor was an effective teacher | 5 | 4.5 | 5 | 4.2 | 5 | 4.9 | 4.3 | | | |
| | | | | | | | | | | |

Response options: 5 = "Strongly Agree", 4 = "Agree", 3 = "Neither Agree/Disagree", 2 = "Disagree", 1 = "Strongly Disagree"

| [Term] [Course] ([Assignment]) | Fal Pectation in | Fal 20 States on In | Fal 200 Matt DOD | Some ausers | standardesol Asia | storestorestorestorestorestorestorestore | and a state of the |
|--|------------------|---------------------|------------------|-------------|-------------------|--|--|
| The instructors teaching aligned with the courses learning objectives/outcomes | 4.8 | 4.8 | 4.5 | 4.8 | 4.8 | 4.5 | |
| The instructor was receptive to students outside the classroom | 4.8 | 4.9 | 4.4 | 4.9 | 4.9 | 4.4 | |
| The instructor explained material well. | 4.8 | 4.8 | 4.1 | 4.5 | 4.6 | 4.2 | |
| The instructor was enthusiastic about teaching the course | 4.9 | 4.9 | 4.4 | 4.7 | 4.5 | 4.4 | |
| The instructor was prepared for class | 4.8 | 4.8 | 4.5 | 4.9 | 4.8 | 4.5 | |
| The instructor gave useful feedback | 4.8 | 4.9 | 4.2 | 4.8 | 4.5 | 4.3 | |
| The instructor consistently treated students with respect | 4.8 | 4.9 | 4.6 | 4.8 | 4.7 | 4.5 | |
| Overall, the instructor was an effective teacher | 4.9 | 4.8 | 4.3 | 4.7 | 4.7 | 4.3 | |

My Evaluations Departmental Average During Term to the Left

Comments on the Strengths and Weaknesses of the Instructor:

Summer 2021-MA131 Calculus for Life and Management Sciences Instructor of Record

- Mr. Bortner was extremely receptive to questions, understood that calculus is a hard subject and took the time to ensure we felt comfortable with the material. Tests also covered exactly what he communicated it would and were adequate measurements of our understanding.
- Cash was an awesome professor! I honestly can't think of an issue that I had with this course that was in his control.
- Overall it was a great course and he was a super helpful professor.
- Cash was an amazing professor. I would recommend his class to anyone.
- Cash was a great instructor. He was passionate about his teaching and emphasized how he wanted all of his students to succeed in his class. He was always open to answering any of our questions and encouraged us to reach out to him.

Spring 2021-MA225 Foundation of Advanced Mathematics Instructor of Record

- I like how Cash is very passionate about helping all students learn and understand the course material. He is very patient and does a great job lifting up the energy during the quiet zoom sessions.
- Cash is a really nice person and a great teacher, seems to go through the material very logically and wants students to succeed. He explains stuff well.
- Was amazingly understanding and was aware of the fact that we have a life out of class. Had so many office hours and was always willing to work through HW with me. I felt as though the tests were fair and covered what we learned fairly well. Overall one of my favorite professors I've had so far.
- He was always there for me and helped me when I was confused.
- The strength of the instructor was that he was very understanding of student's lives, in and out of class, and he knew a lot about the subject.
- Cash was extremely proficient at responding on time and providing helpful information on both the topics that were being discussed and with any extra help that may have been needed. The class also moved at a very solid pace and was easy to follow along with what he was explaining.

Professor Bortner is great about relating to students and remaining approachable throughout the entire semester. This course is very thorough about notation and reasoning, and I found that very helpful in all of my other math courses. And I learned this course better with the amount of proof examples in Cash/Jo-Ann's curriculum. To me, it was easy to ask questions and admit to not knowing everything because Cash created such a welcoming environment. I really liked that he was flexible with due dates, especially if he was behind on grading himself. This made me feel like he was more focused on our well-being and our comprehension of the material, rather than grades or dates that were set early in the semester.

Summer 2019-MA131 Calculus for Life and Management Sciences Instructor of Record

- Was a very good teacher. Explained every question I had.
- He takes the time to explain everything well and is very accessible to students in need of assistance.
- As a student who has always not been a huge fan of math and was terrified even thinking of taking Calculus 1, I've really liked this course. Cash was a great teacher, he was flexible with his office hours and was patient when explaining the concepts sometimes repetitively if I didn't understand it all the first time. He also was very dedicated in creating a review for our class for each exam. The reviews had an outline of all the info that we had learned and an extensive step by step solution page that showed how to solve a problem which was super helpful when studying.

Selected Recitation Instructor Comments

- Cash is amazing at his job. He always responds quickly to emails and tries to help in any way possible. He is really good at explaining material well and how to do problems multiple ways.
- Cash was the main reason I passed this course. I often wish he was the professor for this class as he was great at explaining the information, provided helpful ways to remember information, and had extra office hours whenever he could. He always had a great attitude and was very enthusiastic when helping students.
- He was absolutely fantastic! Hands down the best math teacher I have ever had at State. I honestly learn more from him one day a week then I did all week in class. He made himself super available for office hours and via email for help.

This was invaluable and his teaching is the reason why I have suceeded in this class!

- He did an amazing job and in my opinion, would have been a better choice to teach the class. He was clear and concise when explaining material and always came to class with a great attitude. He is willing to help in any way he can and is committed to the learning of his students. He was always happy to give extra help during office hours and encouraged students to email him with questions. He is also very patient, which I appreciated because it takes me a while to fully grasp some math concepts. Overall, he's a superb instructor and NC State is lucky to have him!
- The best math teacher I've had so far! I'm horrible at math, but Cash made it
 easier to understand for sure. He explained topics very well and I wish I could
 have had recitation more often because that is where I learned the most. I also
 like that he wants you to understand the concepts, and not just memorize them. I
 can't think of any weaknesses!! Rlly a 10/10 calc teacher. Also he had so many
 office hours!! He really wanted us to succeed and you could tell.
- Cashous Bortner was excellent at providing detailed explanations for difficult material, and always encouraged students to ask questions. He created a positive learning environment and was very helpful.
- He is an amazing teacher who not only finds time to always help out students who need it, but also explains the material fabulously. If one explanation doesn't work for a student, he will find another way to help and explain the problems and figure out why the student might be struggling. He was respectful to all students and their questions, and actively involved himself with the students. As someone who typically hates math and can struggle with it, his teaching not only helped but also made me comfortable with math, and I would sign up for his classes again.
- Cash was the best recitation teacher I've ever had. He had such an amazing attitude and drive to help students on calculus.
- Really great at explaining and giving tips to simplify complex problems. Cashous needs to be a teacher he's really good
- I really enjoyed having Cash as my TA for this course. He made me feel very comfortable in the classroom, and I felt that he really cared about our overall knowledge of the subject. He was very receptive to questions, and would also answer them the best that he could (which was typically very well). He was also very responsive to fixing any mistakes that he would make when a student would point them out.

DEPARTMENT OF MATHEMATICS TEACHING EVALUATIONS RESULTS, Fall, 2016 The questions on the questionnaire:

Q11. Was the recitation instructor well prepared?

- J) Nearly always K) Usually L) Frequently M) Seldom N) Almost never
- Q12. Was the recitation instructor effective in answering questions?
 - J) Nearly always K) Usually L) Frequently M) Seldom N) Almost never
- Q13. Did the recitation instructor seem to be interested in helping you? J) Nearly always K) Usually L) Frequently M) Seldom N) Almost never
- Q14. What is your overall impression of how your recitation instructor met his/her responsibilities? J) Excellent K) Very good L) Good M) Fair N) Poor
- Q15. Did the recitation instructor's manner of speaking interfere with effective teaching? J) Never K) Rarely L) Occasionally M) Frequently N) Nearly always

Recitation Instructor **Cashous Bortner**: Course MATH107; Section 354; Enrollment = 24 Response Forms: 9; Response rate: 37.5%; Dept Reci Course Avgs: by student: 59.2%, by course: 59.4%

Avg denotes the weighted average for this class, for each question, where J=4, K=3, etc. for Fall, 2012 to Fall, 2016.

Qrt denotes the quartile of the Avg score with respect to the given comparison group.

DptS denotes the department-wide weighted average of all Student responses for Fall, 2016.

DptC denotes the average over all Courses of the course weighted averages for Fall, 2016.

The quartiles q_1 , q_3 , median m and average \bar{x} are for the distribution of course averages for the given question in the given comparison group. The histograms show relative frequencies,

where bars for group data are open and bars for the current course are solid.

Here is the distribution of responses for this class by question:

| Numbers of responses | | | | | | | percentages | | | | | vg | Qrt | DptS | DptC |
|----------------------|---|---|--------------|---|---|----|-------------|--------------|---|---|---|-----|----------------------|-----------------------|-----------------------|
| | - | | - | | | - | | - | | | | | | | |
| | J | Κ | \mathbf{L} | Μ | Ν | J | K | \mathbf{L} | Μ | Ν | | | | | |
| Q11 : | 8 | 1 | 0 | 0 | 0 | 89 | 11 | 0 | 0 | 0 | 3 | .89 | 4 | 3.35 | 3.37 |
| Q12: | 7 | 2 | 0 | 0 | 0 | 78 | 22 | 0 | 0 | 0 | 3 | .78 | 4 | 3.09 | 3.09 |
| Q13 : | 7 | 1 | 1 | 0 | 0 | 78 | 11 | 11 | 0 | 0 | 3 | .67 | 2 | 3.4 | 3.41 |
| Q14: | 6 | 3 | 0 | 0 | 0 | 67 | 33 | 0 | 0 | 0 | 3 | .67 | 4 | 3.03 | 3.04 |
| Q15: | 6 | 3 | 0 | 0 | 0 | 67 | 33 | 0 | 0 | 0 | 3 | .67 | 3 | 3.34 | 3.33 |
| • | | | | | | | | | | | 1 | | | | |

Histograms for responses for all recitations

(group size: 333 classes, 105 distinct instructors and 4758 response forms for Fall, 2012 to Fall, 2016):



Comments for Recitation Instructor Cashous Bortner: Course MATH107; Section 354; Fall, 2016

Enrollment = 24; Number of Response Forms = 9; Number of comments = 7

Note: Each comment below is from a different student; 2 filled out a survey but did not enter any comment. The numbers in parentheses immediately preceding each comment are that student's numerical responses, provided here to help put the written comment in context.

The comments are in response to the question:

Please comment on the quality of the instruction you received and on your experiences in this course.

- (1) (4, 4, 4, 4, 4): Cash is the best recitation instructor I have. He is extremely helpful and very attentive to his students. He is very good at explaining the topics and making them easy to understand. He is nice and funny and just easy to relate to and talk to about problems. I don't think I would have done nearly as well with any other recitation instructor.
- (2) (4, 4, 4, 3): Cash is a very good instructor. He teaches concepts from the core and pushes for understanding rather than just memorizing. He seems a bit nervous at times, but overall he is extremely gifted at explaining without being condescending in the process.
- (3) (4, 4, 4, 3, 3): Instructor showed a clear passion and understanding of the subject, which translated well to an interactive and helpful recitation
- (4) (4, 4, 4, 4, 4): This guy is awesome. Best TA hands down. He understands the material well and knows how to communicate it to the student. Wish I could have him for all my classes. He knows how to make the student think about the problem while giving helpful hints to progress through the problem. That way the student knows why things happen. Again, he was an awesome TA and will be a great professor in the future.
- (5) (4, 3, 2, 3, 4): I liked having Cash as my instructor because he would go out of his way to help us like when he would hold review sessions based on what times we wanted instead of just picking one and not caring how many of us could make it there. He also understood the material well which a couple of my friends in other recitations said that their TA didn't really help them that much.
- (6) (4, 4, 4, 4, 4): This dude is fucking awesome. Would easily love to have him as my recitation instructor for any other math classes and would easily email him again if in another math and still needed help because he would help anyways.
- (7) (4, 4, 4, 4, 4): Cash was very knowledgeable and willing to help. He truly cares about teaching and helping students.

DEPARTMENT OF MATHEMATICS TEACHING EVALUATIONS RESULTS, Spring, 2016 The questions on the questionnaire:

Q11. Was the recitation instructor well prepared?

- J) Nearly always K) Usually L) Frequently M) Seldom N) Almost never
- Q12. Was the recitation instructor effective in answering questions?
 - J) Nearly always K) Usually L) Frequently M) Seldom N) Almost never
- Q13. Did the recitation instructor seem to be interested in helping you? J) Nearly always K) Usually L) Frequently M) Seldom N) Almost never
- Q14. What is your overall impression of how your recitation instructor met his/her responsibilities? J) Excellent K) Very good L) Good M) Fair N) Poor
- Q15. Did the recitation instructor's manner of speaking interfere with effective teaching? J) Never K) Rarely L) Occasionally M) Frequently N) Nearly always

Recitation Instructor **Cashous Bortner**: Course MATH106; Section 351; Enrollment = 20 Response Forms: 17; Response rate: 85%; Dept Reci Course Avgs: by student: 69.6%, by course: 69.2%

Avg denotes the weighted average for this class, for each question, where J=4, K=3, etc. for Fall, 2012 to Spring, 2016.

Qrt denotes the quartile of the Avg score with respect to the given comparison group.

DptS denotes the department-wide weighted average of all Student responses for Spring, 2016.

DptC denotes the average over all Courses of the course weighted averages for Spring, 2016.

The quartiles q_1 , q_3 , median m and average \bar{x} are for the distribution of course averages for the given question in the given comparison group. The histograms show relative frequencies,

where bars for group data are open and bars for the current course are solid.

Here is the distribution of responses for this class by question:

| Numbers of responses | | | | | | | | percentages | | | | g Qr | t DptS | DptC |
|----------------------|----|---|---|---|---|-----|---|-------------|---|---|------|------|--------|-----------------------|
| | _ | | _ | | | _ | | _ | | | | | | |
| | J | Κ | L | Μ | Ν | J | Κ | L | Μ | Ν | | | | |
| Q11 : | 16 | 1 | 0 | 0 | 0 | 94 | 6 | 0 | 0 | 0 | 3.94 | 4 4 | 3.51 | 3.45 |
| Q12: | 16 | 1 | 0 | 0 | 0 | 94 | 6 | 0 | 0 | 0 | 3.94 | 4 4 | 3.35 | 3.28 |
| Q13 : | 17 | 0 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 4 | 4 | 3.6 | 3.55 |
| Q14 : | 17 | 0 | 0 | 0 | 0 | 100 | 0 | 0 | 0 | 0 | 4 | 4 | 3.32 | 3.24 |
| Q15: | 16 | 1 | 0 | 0 | 0 | 94 | 6 | 0 | 0 | 0 | 3.94 | 4 4 | 3.44 | 3.41 |
| | | | | | | | | | | | | | | |

Histograms for responses for all recitations

(group size: 333 classes, 105 distinct instructors and 4758 response forms for Fall, 2012 to Spring, 2016):

| Quest 11 | Quest 12 | Quest 13 | Quest 14 | Quest 15 |
|------------------|------------------|------------------|------------------|----------------|
| $\bar{x} \ 3.39$ | $\bar{x} \ 3.28$ | $\bar{x} \ 3.57$ | $\bar{x} \ 3.19$ | \bar{x} 3.35 |
| $q_1 \ 3.16$ | $q_1 \ 3$ | $q_1 \ 3.44$ | $q_1 \ 2.89$ | $q_1 \ 3.14$ |
| $m \ 3.5$ | $m \ 3.43$ | m 3.7 | $m \ 3.32$ | $m \ 3.5$ |
| $q_3 3.75$ | $q_3 3.71$ | $q_3 3.86$ | $q_3 3.65$ | $q_3 3.75$ |

Comments for Recitation Instructor Cashous Bortner: Course MATH106; Section 351; Spring, 2016

Enrollment = 20; Number of Response Forms = 17; Number of comments = 13

Note: Each comment below is from a different student; 4 filled out a survey but did not enter any comment. The numbers in parentheses immediately preceding each comment are that student's numerical responses, provided here to help put the written comment in context.

The comments are in response to the question:

Please comment on the quality of the instruction you received and on your experiences in this course.

- (1) (4, 4, 4, 4, 4): Great teacher, very helpful in understanding harder concepts and giving examples of them. Teaching in a way student can relate to a little better. Big part of my learning in this course.
- (2) (4, 4, 4, 4, 4): Best recitation instructor i've had. He was willing to go out of his way to help us. I'd like to have a beer with the guy.
- (3) (4, 4, 4, 4, 4): Cash is helpful. One time our group stuck the team homework and I emailed him lots of times, he helped me step by step until I found the way to solve the problem.
- (4) (4, 4, 4, 4, 4): Cash was always prepared, taught everything very well, always helped when we needed it, and if we didn't get something, he would walk us through it and explain it until we understood.
- (5) (4, 4, 4, 4, 4): Absolutely wonderful. Really offers a lot to us students and shows that he wants us to get math and succeed. I would definitely take a recitation with him again!
- (6) (4, 4, 4, 4, 4): Very helpful and always prepared. His review sessions before the exam always helped and he encouraged students to come to his office hours which just showed how much he wants this section to succeed in the class.
- (7) (3, 3, 4, 4, 3): Everything I learned was from Cashous. recitation was were everything was explained better. If lecture was more like recitation, I think things would be easier for people to grasp.
- (8) (4, 4, 4, 4, 4): Cash was a great TA. He helped us out a ton. Very helpful and knew his stuff.
- (9) (4, 4, 4, 4, 4): AMAZING! I would not have done as well in Calculus without the help of Cashous. He definitely knows what he is talking about and is ALWAYS willing to help. Will meet up, responds quickly to e-mails, and answers everything you ask. Highly recommend!
- (10) (4, 4, 4, 4, 4): Always worked problems out on the broad AND went to every student to help individually. LOVED IT!!
- (11) (4, 4, 4, 4, 4): Amazing T.A. for recitation. Seemed very smart, always answered questions quickly.
- (12) (4, 4, 4, 4, 4): Great teacher with high interest!
- (13) (4, 4, 4, 4, 4): Hi

DEPARTMENT OF MATHEMATICS TEACHING EVALUATIONS RESULTS, Fall, 2015 The questions on the questionnaire:

- Q11. Was the recitation instructor well prepared?
- J) Nearly always K) Usually L) Frequently M) Seldom N) Almost never
- Q12. Was the recitation instructor effective in answering questions?
 - J) Nearly always K) Usually L) Frequently M) Seldom N) Almost never
- Q13. Did the recitation instructor seem to be interested in helping you? J) Nearly always K) Usually L) Frequently M) Seldom N) Almost never
- Q14. What is your overall impression of how your recitation instructor met his/her responsibilities? J) Excellent K) Very good L) Good M) Fair N) Poor
- Q15. Did the recitation instructor's manner of speaking interfere with effective teaching? J) Never K) Rarely L) Occasionally M) Frequently N) Nearly always

Recitation Instructor **Cashous Bortner**: Course MATH106; Section 251; Enrollment = 25 Response Forms: 16; Response rate: 64%; Dept Reci Course Avgs: by student: 55.3%, by course: 55.9%

Avg denotes the weighted average for this class, for each question, where J=4, K=3, etc. for Spring, 2013 to Fall, 2015.

Qrt denotes the quartile of the Avg score with respect to the given comparison group.

DptS denotes the department-wide weighted average of all Student responses for Fall, 2015.

DptC denotes the average over all Courses of the course weighted averages for Fall, 2015.

The quartiles q_1 , q_3 , median m and average \bar{x} are for the distribution of course averages for the given question in the given comparison group. The histograms show relative frequencies,

where bars for group data are open and bars for the current course are solid.

Here is the distribution of responses for this class by question:

| Nı | Numbers of responses | | | | | | | | entag | ges | Avg | Qrt | DptS | DptC |
|-------|----------------------|------------|--------------|---|---|----|----|---|-------|-----|------|----------------------|-----------------------|-----------------------|
| | - | T 7 | Ŧ | | | - | | Ŧ | | | | | | |
| | J | Κ | \mathbf{L} | Μ | Ν | J | K | L | Μ | Ν | | | | |
| Q11 : | 15 | 1 | 0 | 0 | 0 | 94 | 6 | 0 | 0 | 0 | 3.94 | 4 | 3.4 | 3.41 |
| Q12: | 14 | 2 | 0 | 0 | 0 | 88 | 13 | 0 | 0 | 0 | 3.88 | 4 | 3.34 | 3.35 |
| Q13 : | 14 | 1 | 1 | 0 | 0 | 88 | 6 | 6 | 0 | 0 | 3.81 | 3 | 3.6 | 3.58 |
| Q14 : | 15 | 1 | 0 | 0 | 0 | 94 | 6 | 0 | 0 | 0 | 3.94 | 4 | 3.23 | 3.23 |
| Q15: | 15 | 1 | 0 | 0 | 0 | 94 | 6 | 0 | 0 | 0 | 3.94 | 4 | 3.44 | 3.43 |

Histograms for responses for all recitations

(group size: 250 classes, 97 distinct instructors and 3456 response forms for Spring, 2013 to Fall, 2015):

| Quest 11 | Quest 12 | Quest 13 | Quest 14 | Quest 15 |
|---------------|---------------|------------------|---------------|----------------|
| $\bar{x} 3.4$ | \bar{x} 3.3 | $\bar{x} \ 3.58$ | $\bar{x} 3.2$ | \bar{x} 3.34 |
| $q_1 \ 3.18$ | $q_1 \ 3$ | $q_1 \ 3.44$ | $q_1 \ 2.9$ | $q_1 \ 3.13$ |
| $m \ 3.5$ | $m \ 3.5$ | m 3.7 | $m \ 3.33$ | $m \ 3.5$ |
| $q_3 3.73$ | $q_3 3.71$ | $q_3 3.88$ | $q_3 3.67$ | $q_3 3.76$ |

Comments for Recitation Instructor Cashous Bortner: Course MATH106; Section 251; Fall, 2015

Enrollment = 25; Number of Response Forms = 16; Number of comments = 11

Note: Each comment below is from a different student; 5 filled out a survey but did not enter any comment. The numbers in parentheses immediately preceding each comment are that student's numerical responses, provided here to help put the written comment in context.

The comments are in response to the question:

Please comment on the quality of the instruction you received and on your experiences in this course.

- (1) (4, 4, 4, 4, 4): The instructor was immensely helpful and I strongly feel that without him and his motivation to do everything he could to help the students learn the material I would not have done as well in the class.
- (2) (4, 4, 4, 4, 4): Cash is a very good recitation TA. He takes time to hold review sessions outside of class before exams.
- (3) (4, 4, 4, 4, 4): I fully relied on Cash for my grade in this class, and I am so thankful to have had him as a TA. He would always make himself available if a student needed help, and genuinely cared about is student's grades, concerns, and questions on the subject at hand. I would have a terrible grade in the course if it were not for how easy he is to understand. He shows us easier ways to solve problems, and essentially just wants us to succeed the best that we can.
- (4) (4, 4, 4, 4, 4): Cash was a really great recitation TA. He laid things out and made them really easy to understand. He answered all of our questions really well and in a way that didn't make us feel stupid for asking them. I think I learned more in recitation than I did in lecture. Cash also went above and beyond just recitation - he offered us review sessions before every exam and rewarded us for doing well.
- (5) (4, 4, 4, 4, 4): Cash was extremely helpful and often went out of his way to make a relaxed and open environment. He helped us with extra study sessions outside of class time and was very open and helpful with any questions we had.
- (6) (4, 4, 4, 4, 3): Cashous is amazing. For a student with so many other responsibilities, he seemed to really care about all of us. He took time out of his schedule to plan study sessions before tests, he seemed well prepared to teach us every morning and was enthusiastic even in an 8:30 am class. He was honest about things being difficult but he stayed positive and encouraged all of us that we could succeed. Cashous is defiantly the reason I succeeded in calculus!
- (7) (4, 4, 4, 4, 4): Cashous was amazing. He was very helpful. I would not have passed the course without him.
- (8) (4, 4, 4, 4, 4): Cash explains things in a simple manner that helps a lot. And he host two hour study group's the night before the test to help us. KEEP HIM, HE IS AWESOME.
- (9) (4, 4, 4, 4, 4): Cash was always more than willing to provide help to myself and all the students in his recitation class. Whenever I emailed him with questions he was quick to respond with assistance. He would even have a review session before each test. Given the smaller class sizes and the effective teaching, I probably learned more from this recitation than I did from the class itself.
- (10) (4, 4, 4, 4, 4): Amazing job, went above and beyond the required responsibilities. Best ta I've ever had.
- (11) (4, 4, 4, 4, 4): great, He really cares and is good at explaining.

MA 225 (002) - Foundations of Advanced Mathematics

Spring 2021

| Instructor: | Cash Bortner (he/him/his) | Time: | MWF 11:45 – 12:35pm |
|-------------|---------------------------|------------|------------------------------------|
| Email: | cwbortne@ncsu.edu | Zoom Link: | https://ncsu.zoom.us/j/95346536979 |

Office Hours: MWF 12:35-1:35pm via Zoom Link https://ncsu.zoom.us/j/95346536979 and by appointment.

Prerequisites: MA 241

GEP Category: This course does not fulfill a General Education Program requirement.

Materials/book: The notes are on the course website. (There are no additional charges for the book for the course.)

Course Description: Introduction to mathematical proof with focus on properties of the real number system. Elementary symbolic logic, mathematical induction, algebra of sets, relations, functions, countability. Algebraic and completeness properties for reals.

Student Learning Objectives/Outcomes: Students should demonstrate the ability to:

- 1. read, understand, and make informed judgments about mathematical arguments.
 - Students should be able to identify the basic methods of proof such as direct proofs, proofs by contradiction, and proofs using the Principle of Mathematical Induction and equivalent principles.
 - Students should be able to analyze and critique mathematical arguments. Students should be able to make a sound case as to the correctness of the argument, should be able to explain any problems that are found, and then should be able to suggest ways to remedy the problems.
- 2. generate clearly reasoned, convincing proofs.
 - Students should be able to construct proofs using direct arguments, contradiction, and the Principle of Mathematical Induction and equivalent principles.
 - Students should be able to move successfully from the discovery of a proof to the writing of a proof.
 - Students should be able to construct a proof in such a way that the reader can follow the steps logically from one to another and is convinced of the validity of the proof.
 - Students should be able to show that they possess a sound understanding of the underlying mathematics as revealed in the construction of proofs.
- 3. clearly communicate mathematics (in both written form and oral form) at the appropriate levels for the audience.
- 4. understand and use mathematical language and symbols.
 - Students should be able to express mathematical concepts using precise mathematical symbols and prose.
- 5. explore examples and make conjectures.

- 6. understand the basic definitions and properties of selected mathematical concepts beyond calculus and high school material.
 - Students should be able to use these definitions and properties in the construction of proofs.

Course Structure and Rules of Engagement: The lectures will be conducted synchronously via Zoom and will be interspersed with small group activities. The course will be structured so that there is a lot of student interaction. In this class we will share ideas and learn from each other. It will be important for each of you to "try" on new ideas and to make conjectures even if you are not sure that you are correct. You will also be asked to critique the ideas and arguments of others. While it is OK to disagree with those ideas and arguments, it is not OK to make personal attacks.

Course Delivery Changes Related to COVID-19: Please be aware that the situation regarding COVID-19 is frequently changing, and the delivery mode of this course may need to change accordingly, including possibly moving from synchronous sessions to an asynchronous format. Regardless of the delivery method, we will all strive to provide a high-quality learning experience.

Note: If I need to be out for an extended period of time, Dr. Jo-Ann Cohen will take over in my absence.

Email: All emails will be sent to your official ncsu.edu email address.

Course Schedule: I will usually assign homework each week and will collect homework every 7-10 days. The topics we will cover and the order in which we will cover them are listed below. The pace of the course depends upon your interests and input. The flexibility in the course topics schedule enables us to pursue your ideas, your conjectures, and your proofs.

- Chapter 0. Preliminaries: 1-2 days
- Chapter 1. Language, Logic, Sets, and Methods of Proofs: 3-5 weeks
- Chapter 2. The Principle of Mathematical Induction: 2-3 weeks
- Chapter 3. Functions: 2-3 weeks
- Chapter 4. Equivalence Relations: 2-3 weeks
- Chapter 5. The Size of Sets: 1-2 weeks

Chapter 6. Further Properties of the Integers: 1 week (if time)

Note that the above course schedule is subject to change.

Wellness Days: The wellness days for Spring 2021 are:

- Tuesday, February 9th,
- Friday, March 5th,
- Wednesday, March 24th,
- Thursday, April 15th.

We will not have class on those days, and will not have tests or assignments due on any of those days.

Grading:

| 1. | Homework Assignments | 10% |
|----|--|-----|
| 2. | Three Hour Exams $(20\% \text{ each})$ | 60% |
| | Tentative Dates: | |
| | Wednesday, February 17th | |
| | Monday, March 22nd | |
| | Friday, April 23rd | |
| 3. | Final Exam | 30% |

Monday, May 3rd, 12:00-2:30 p.m.

Standard Grading Scale: I will use the standard grading scale:

| $97 \le A + \le 100,$ | $93 \le A < 97,$ | $90 \le A - < 93$, |
|-----------------------------|------------------|---------------------|
| $87 \le B + < 90,$ | $83 \le B < 87,$ | $80 \le B - < 83,$ |
| $77 \le C + < 80,$ | $73 \le C < 77,$ | $70 \le C - < 73,$ |
| $67 \le \mathrm{D} + < 70,$ | $63 \le D < 67,$ | $60 \le D- < 63,$ |
| $F \leq 60.$ | | |

Grading/Scheduling Changing Options Related to COVID-19: If the delivery mode has a negative impact on your academic performance in this course, the university has provided tools to potentially reduce the impact:

- Enhanced S/U Grading Option: Enhance Satisfactory/Unsatisfactory Grading Option
- Late Drop: Enhanced Late Drop Option

Be aware that if you use the enhanced S/U grading option, you will need to complete the course and receive at least a C- to pass the course.

In some cases, another option may be to request an incomplete in the course. Before using any of these tools, please discuss options with me and with your academic advisor.

Auditing the Course: To audit the course, you must have the approval of your advisor and the Mathematics Department. In order to receive an AU, you must attend the majority of the sessions, and you must hand in all of the homework and take all of the tests. See:

https://policies.ncsu.edu/regulation/reg-02-20-04/

for more information.

Incomplete Grades: Incomplete grades will be handled on an individual basis. Note, however, that if an extended deadline is not authorized by an instructor or department, an unfinished incomplete grade will automatically change to an F after either (a) the end of the next regular semester in which the student is enrolled (not including summer sessions), or (b) the end of 12 months if the student is not enrolled, whichever is shorter. Incomplete grade to F will count as attempted courses on transcripts. The burden of fulfilling an incomplete grade is the responsibility of the student. The university policy on incomplete grades is located at:

https://policies.ncsu.edu/regulation/reg-02-50-03/

Attendance Policy/Late Assignments/Make-up Work: You are expected to attend all classes on time. Late assignments and make-up work will only be allowed for excused absences. You should contact me by email before any anticipated excused absence. If you have an unanticipated excused absence (for example, a medical emergency), you should contact me within one week of returning to class. For complete attendance and excused absence policies, please see:

https://policies.ncsu.edu/regulation/reg-02-20-03-attendance-regulations/

COVID-19 Related Absences: If you need to miss class because you have been advised that you may have been exposed to COVID-19 or you have a personal or family situation related to COVID-19 that prevents you from attending our sessions, please contact me. Together we will develop a plan to help you keep up with your coursework during any such absences. COVID 19-related absences will be considered excused. You do not need any additional documentation. (But, again, please make sure you contact me.)

Academic Integrity/Honesty: Students are required to comply with the university policy on academic integrity/honesty found in the Code of Student Conduct:

https://policies.ncsu.edu/policy/pol-11-35-01/

It is my understanding and expectation that your signature on any test or assignment means that you have adhered to the Pack Pledge:

I have neither given nor received unauthorized aid on this test or assignment.

Violations of academic integrity will be handled in accordance with the Student Discipline Procedures (NCSU REG 11.35.02).

Digital Course Components: In this course we will use Zoom and Moodle. Please see the relevant technology requirements. If you need access to additional technological support, please contact the Libraries' Technology Lending Service (Technology Lending).

Students may be required to disclose personally identifiable information to other students in the course, via electronic tools like email or web-postings, where relevant to the course. Examples include online discussions of class topics and posting of student coursework. All students are expected to respect the privacy of each other by not sharing or using such information outside the course.

Moodle: All reading materials are housed on the course website. Note that I received licenses to include pictures and biographies of the mathematicians and mathematics educators that appear in the notes under the provision that the materials would be housed on a secure site. So although you may copy the notes for your own use, you should not share the pages with the pictures and biographies with anyone else. Throughout the semester you will need to upload homework and tests in Moodle as pdfs. So make sure you

Throughout the semester you will need to upload homework and tests in Moodle as pdfs. So make sure you have an app that will allow you to scan your work if you are not able to directly save your work as a pdf.

Zoom: Our "lectures" will be conducted synchronously (Monday, Wednesday, and Friday from 11:45 a.m. to 12:35 p.m.) via Zoom. For the first 1-2 weeks, I will send out a Zoom invitation to your NC State email address right before class and you will be able to click on the link in that email. After that, I will expect you to access the link below for our meetings:

https://ncsu.zoom.us/j/95346536979

I will initially put all of you on "mute", but if you want to talk, there is a button on the bottom-left side of the Zoom screen that will enable you to unmute yourself. Zoom enables us to have breakout rooms for small group discussions and we will utilize that function as well.

I will be recording our Zoom sessions and will be posting links to the recordings on our Moodle page for your use. I will only record our mathematical discussions and not, for example, the parts of our sessions where we check in with each other. In order that all students in the class feel comfortable asking questions, and making conjectures, etc., you may not share the links to the recordings with anyone outside of our class. These recordings are for use in our current class (and possibly for use in future educational activities). By your continued participation in this recorded course, you are providing your permission to be recorded for use in our class and for use in future educational activities.

Diversity, Equity, and Inclusion: Diversity, equity, and inclusion are important to the success of our students at NC State. Every student, every faculty member, and every staff member who comes to NC

State enriches us through their varied perspectives, knowledge, and backgrounds. Our classroom is one in which every student is respected and feels heard.

In an effort to affirm and respect the identities of transgender students in the classroom and beyond, please contact me if you wish to be referred to using a name and/or pronouns other than those listed in the student directory.

I have benefited throughout my life from my majority status, the most impactful of which has been my white privilege. I know that our students of color face issues that I have never faced, nor will ever face. I want to affirm that I will listen to your experiences and to the experiences of all of my students if you would like to share them with me. I value your perspectives and I will advocate for your needs in our department and throughout the university.

I welcome any additional suggestions you have for making our classroom more welcoming and inclusive.

Accommodations for Disabilities: Reasonable accommodations will be made for students with verifiable disabilities. In order to take advantage of available accommodations, students must register with the Disability Resource Office at Holmes Hall, Suite 304, 2751 Cates Avenue, Campus Box 7509, 919-515-7653. For more information on NC State's policy on working with students with disabilities, please see the Academic Accommodations for Students with Disabilities Regulation (REG02.20.01).

Non-Discrimination Policy: NC State prohibits discrimination, harassment, and retaliation based on a person's age (40 years or older), color, disability, genetic information, gender identity, national origin, race, religion, sex (including pregnancy), sexual orientation or veteran status. If you feel that you have been the subject of prohibited discrimination, harassment, or retaliation, you should contact the Office for Institutional Equity and Diversity (OIED) at 919-513-0574.

NC State's policies and regulations covering discrimination, harassment, and retaliation may be accessed at http://policies.ncsu.edu/policy/pol-04-25-05/ or http://oied.ncsu.edu/divweb.

Health and Well-Being Resources: These are difficult times, and academic and personal stress are natural results. Everyone is encouraged to take care of themselves and their peers. If you need additional support, there are many resources on campus to help you:

- Counseling Center (NCSU Counseling Center)
- Health Center (Health Services Student)
- NC State CARES Team: As members of the NC State Wolfpack community, we each share a personal responsibility to express concern for one another and to ensure that our campus remains a healthy and safe environment for learning. Occasionally, you may come across a classmate whose personal behavior concerns or worries you, either for your classmate's well-being, for your well-being, or for the well-being of others. When this is the case, I would encourage you to report the behavior to the NC State CARES team: (Share a Concern).
- If you or someone you know are experiencing food, housing or financial insecurity, please see the Pack Essentials Program (Pack Essentials).

 $\label{eq:actional} \begin{array}{l} \textbf{Additional COVID-19 Information:} \end{tabular} \end{tabu$

We are most concerned about your health and the health of the students, faculty, and staff across campus. If you test positive for COVID-19, or are told by a healthcare provider that you are presumed positive for the virus, please follow university guidelines, including self- reporting (Coronavirus Self Reporting): Self-reporting is not only to help provide support to you, but also to assist in contact tracing for containing the spread of the virus.

Community Standards related to COVID-19: We are all responsible for protecting ourselves and our

community. Please see the Community Standards and Rule 04.21.01 regarding Personal Safety Requirements Related to COVID-19 (RUL 04.21.01 – Personal Safety Requirements Related to COVID-19 – Policies, Regulations & Rules).

NC State Rules and Regulations: Students are responsible for reviewing the NC State University Policies, Rules, and Regulations (PRRs) which pertain to their course rights and responsibilities, including those referenced both below and above in this syllabus:

- Equal Opportunity and Non-Discrimination Policy Statement https://policies.ncsu.edu/policy/pol-04-25-05/ with additional references at https://oied.ncsu.edu/equity/policies/
- Code of Student Conduct https://policies.ncsu.edu/policy/pol-11-35-01/

Important Resources for Students

- NC State Keep Learning, tips for students taking courses remotely: https://dasa.ncsu.edu/academics/keep-learning/
- Introduction to Zoom for Students: https://youtu.be/5LbPzzPbYEw
- Learning with Moodle, a student's guide to using Moodle: https://moodle-projects.wolfware.ncsu.edu/course/vi
- Protect the Pack FAQs: https://www.ncsu.edu/coronavirus/frequently-asked- questions/
- NC State Protect the Pack Resources for Students: https://www.ncsu.edu/coronavirus/reactivating-campus/resources-for-students/

Exam 3 Due: Saturday, April 24th at 10:00pm

This exam is due on Saturday, April 24th at 10:00pm. There will be no late penalty if the exam is turned in by Monday, April 26th by 11:00am. For this exam, you will be working independently. You will submit your work via gradescope 4.5 hours after you started this exam.

You are permitted to use any of your notes from this course, homework assignments from this course, and anything on the Moodle page for this course. You are **NOT** permitted to use any online resources outside of the Moodle page, or talk to each other about the exam.

As always, you will need to **justify your answers**.

You should include, and sign, the Pack Pledge on your test:

I have neither given nor received unauthorized aid on this test.

1. (33 points) Define
$$f = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y = x^2 - 1\}$$

- (a) (5 points) Show that f is a function. Solution: Suppose there exists $(a,b) \in f$ and $(a,c) \in f$. Then we have that $b = a^2 + 1$ and $c = a^2 + 1$ by the definition of f. Therefore, b = c, thus f is a function.
- (b) (7 points) Is f onto \mathbb{R} ? Prove your claim.

Solution: No. The function f is not onto \mathbb{R} . Note that for any element $x \in \mathbb{R}$, then $f(x) = x^2 - 1$. Since the square of any real number is nonnegative, then $f(x) = x^2 - 1 \ge 0 - 1 = -1$, thus $\operatorname{Rng}(f) \subseteq [-1, \infty)$. Therefore, the range of f is not the real numbers, as it cannot contain any real number smaller than -1, i.e. f is **not** onto \mathbb{R} .

In particular, consider y = -2. If we suppose for the sake of contradiction that there exists some $x \in \text{Dom}(f) = \mathbb{R}$ such that f(x) = -2, then we would have that $x^2 - 1 = -2$, hence $x^2 = -1$. Because x is a real number, then $x^2 \ge 0$, a contradiction. Thus, -2 is a real number which is not in the range of f.

- (c) (7 points) Is f injective? Prove your claim. Solution: No. The function f is not injective. Note that $(1,0) \in f$ since $0 = 1^2 - 1$, and similarly $(-1,0) \in f$ as $(0 = (-1)^2 - 1$, however $1 \neq -1$. Therefore, f is not injective.
- (d) (7 points) Find f^{-1} . Write in the form $f^{-1} = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y = \cdots \}$. (Be sure to put all of your steps when finding f^{-1} .)

Solution: By definition,

$$f^{-1} = \{(u, v) \in \mathbb{R} \times \mathbb{R} \colon (v, u) \in f\}$$

= $\{(u, v) \in \mathbb{R} \times \mathbb{R} \colon u = v^2 - 1\}$
= $\{(u, v) \in \mathbb{R} \times \mathbb{R} \colon v^2 = u + 1\}$
= $\{(u, v) \in \mathbb{R} \times \mathbb{R} \colon v = \pm \sqrt{u + 1}\}$
= $\{(x, y) \in \mathbb{R} \times \mathbb{R} \colon y = \pm \sqrt{u + 1}\}$

- (e) (7 points) Is f⁻¹ a function? Prove your claim. Solution: No. Since f is not injective, f⁻¹ is not a function by the Theorem in the Chapter 3: Injections worksheet. More explicitly, (0,1) ∈ f⁻¹ since 1 = √0+1 and similarly (0,-1) ∈ f⁻¹ since -1 = -√0+1, however 1 ≠ -1, thus f⁻¹ is not a function.
- 2. (34 points) Define the **equivalence relation** S on the set of integers \mathbb{Z} as

 $S = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \colon 2 \text{ divides } a + b\}.$

- (a) (10 points) Show that S is reflexive.
 Solution: Suppose a ∈ Z. Note then that a + a = 2a, hence a + a can be written as the product of 2 and an integer, namely a, thus a + a is divisible by 2. Therefore, (a, a) ∈ S for all a ∈ Z, i.e. S is reflexive.
- (b) (10 points) Show that S is symmetric.

Solution: Suppose $(a, b) \in S$. By definition, a + b must be divisible by 2, i.e. there exists some integer k such that a + b = 2k. Note that because the integers are commutative with respect to addition, we have that a + b = b + a, thus b + a = 2k. Since b + a can be written as the product of 2 and an integer, then it is divisible by 2. Thus, $(b, a) \in S$ whenever $(a, b) \in S$, i.e. S is symmetric.

(c) (10 points) Show that S is transitive.

Solution: Suppose $(a, b) \in S$ and $(b, c) \in S$. Then, by definition, a + b = 2k and b + c = 2j for some integers k and j. If we add these two equations we get a + b + b + c = 2k + 2j, which is equivalent to

$$a + c = 2(k + j - b).$$

Since k, j and b are all integers, then so too is j + k - b by the closure property of integers, thus a + c can be written as the product of 2 and an integer. Therefore, $(a, c) \in S$ whenever $(a, b) \in S$ and $(b, c) \in S$, i.e. S is transitive.

(d) (4 points) Find the equivalence class [1] for S.Solution: By definition, the equivalence class of 1 is

$$[1] = \{x \in \mathbb{Z} : (1, x) \in S\} \\= \{x \in \mathbb{Z} : 2 \text{ divides } 1 + x\} \\= \{x \in \mathbb{Z} : 1 + x = 2k \text{ for some } k \in \mathbb{Z}\} \\= \{x \in \mathbb{Z} : x = 2k - 1 \text{ for some } k \in \mathbb{Z}\} \\= \{\dots, -5, -3, -1, 1, 3, 5, \dots\} \\= \{x \in \mathbb{Z} : x \text{ is odd }\}$$

Thus, the equivalence class of [1] is the set of odd integers.

3. (33 points) Define the relation R defined on the set of integers \mathbb{Z} as

$$R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \colon ab \ge 0\}.$$

- (a) (10 points) Is the relation R reflexive? Prove your claim. Solution: Yes. Suppose $a \in Z$, then $a \cdot a = a^2 \ge 0$, thus $(a, a) \in R$. Therefore, R is reflexive.
- (b) (10 points) Is the relation R symmetric? Prove your claim.
 Solution: Yes. Suppose (a, b) ∈ R, then by definition ab ≥ 0. Since multiplication over the integers is commutative, we know that ab = ba, so since ab ≥ 0 so to ba ≥ 0. Therefore, (b, a) ∈ R whenever (a, b) ∈ R, i.e. R is symmetric.
- (c) (10 points) Show that the relation R is not transitive. Solution: No. As a counterexample, consider $(-1,0) \in R$ since $-1 \cdot 0 = 0 \ge 0$, and $(0,2) \in R$ since $0 \cdot 2 = 0 \ge 0$. Note then that $(-1,2) \notin R$ since $-1 \cdot 2 = -2 \ge 0$. Therefore, R is not transitive.